Technical note on definition of zenith total delay retrieval

1 Calculation of zenith total delay

Atmospheric zenith total delay (*ZTD*) is defined as:

$$ZTD = ZHD + ZWD, \quad [\mathrm{m}] \quad (1)$$

where the first term of the equation 1 represents the zenith hydrostatic delay (*ZHD*) and the 2nd term of equation 1 is defined as the wet delay (ZWD). The first term of the equation 1 *ZHD* can be derived as:

$$ZHD = 10^{-6} \cdot k_1 \cdot 0.01 \cdot \frac{R}{m_d} \int_{h_0}^{TOA} \rho(z)\, dz, \quad [\mathrm{m}] \quad (2)$$

where $\mathrm{h_0}$ is a height of the receiver above the geoid, *R* is a gas constant (*R* = 8.31434 ± 0.35 J/mol K), $\mathrm{m_d}$ is a molar mass of dry air ($\mathrm{m_d}$ = 0.0289644 ± 0.35 kg/mol), $\int_{\mathrm{h_0}}^{\mathrm{TOA}} \rho(z)\, dz$ is the integral extend between the height $\mathrm{h_0}$, where the receiver is located and the top of the atmosphere (TOA), $\rho(z)$ is an air density profile, k_1 is an air refractivity coefficient of ”best average” Rueger 2002 [1,2] in Table 1.

Table 1 – Air refractivity coefficients of ”best average” Rueger 2002 [1,2]

Author(s)	k_1, K/hPa	k_2 , K/hPa	$k_3 \cdot 10^{-5}$, K²/hPa	k_2'' , K/hPa
Rueger - ”best average” 2002	77.689 ± 0.0094	71.2952 ± 1.3	3.75463 ± 0.0076	22.99

The 2nd term of the equation 1 *ZWD* is defined as:

$$ZWD = 10^{-6} \cdot \int_{h_0}^{TOA} \left(k_2'' \frac{e}{T} Z_v^{-1} + k_3 \frac{e}{T^2} Z_v^{-1} \right) dz, \quad [\mathrm{m}] \quad (3)$$

where e [hPa] is water vapour pressure, T is temperature [K], Z_d and Z_v are the compressibility factors of dry air and water vapour, a coefficient $k_2'' = k_2 - k_1 \frac{m_v}{m_d}$, where $\frac{m_v}{m_d} = 0.62197732$ and $\mathrm{m_v}$ is molar mass of water vapour ($\mathrm{m_v} = 0.0180152 \pm 0.35\ \mathrm{kg/mol}$). The values of the coefficients k_2'' and k_3 are given in Table 1.

Compressibility of a dry air is defined as: $Z_d^{-1} = 1 + P_d \left[57.90 \cdot 10^{-8} \left(1 + \frac{0.52}{T}\right) - 9.4611 \cdot 10^{-4} \frac{t}{T^2} \right]$, (4)

where P_d is a dry air pressure [hPa], t is an air temperature [°C].

Compressibility of water vapour is defined as:

$$Z_v^{-1} = 1 + 1650 \left(\frac{P_w}{T^3}\right) (1 - 0.01317t + 1.75 \cdot 10^{-4} t^2 + 1.44 \cdot 10^{-6} t^3) \quad (5)$$

2 Retrieval accuracy and self-test

The retrievals require input data of at least 25000 profiles of radiosondes and/or re-analysis profiles. The performance of the retrievals depends on the quality of the input data and its representativeness with respect to the location site. The height match and a good coverage of typical atmospheric states is important to gain the best possible statistical relation between observations and retrieved atmospheric parameters. The retrievals are based on Neural Network (NN) algorithm. These non-linear NN need to be trained using large data sets (~15000) of vertical profile information (temperature, humidity, pressure and liquid water content). During the training the wet and dry delays and corresponding simulated brightness temperatures (via a radiative transfer model) are calculated and used to derive the coefficients of the NN. The remaining part of the profiles which are not used for the training is needed for the self-test of the retrievals. For each set of the simulated brightness temperatures the NN coefficients are applied to retrieve wet and dry delays. The delay retrievals are ‘multi-angular’ which means that they are applied to different 19 elevation angles and have a separate set of NN coefficients for each elevation angle. As an example, the result of the self-test of the retrieved wet and dry delays for zenith are shown on the Figure1. The comparison of the ‘true delay’ and ‘retrieved delay’ gives the correlation, RMSE and a bias. Table 2 summarized the results of the self-test.

For wet delay of the HATPRO retrieval at Meckenheim the root mean squared error is 1.75 mm. The correlation of the wet delay between the ‘true’ and ‘retrieved’ values is 0.999. The relative error of wet delay for values in the range from 5 to 25 cm is ~2 %, the relative error is higher ~9 % for a relatively low values of wet delay < 5 cm. For dry delay the RMSE is 4.24 mm and the correlation coefficient is 0.981. The accuracy of the HATPRO retrieval for dry delay in the range 2.0 - 2.3 m is up to ~0.5 %.

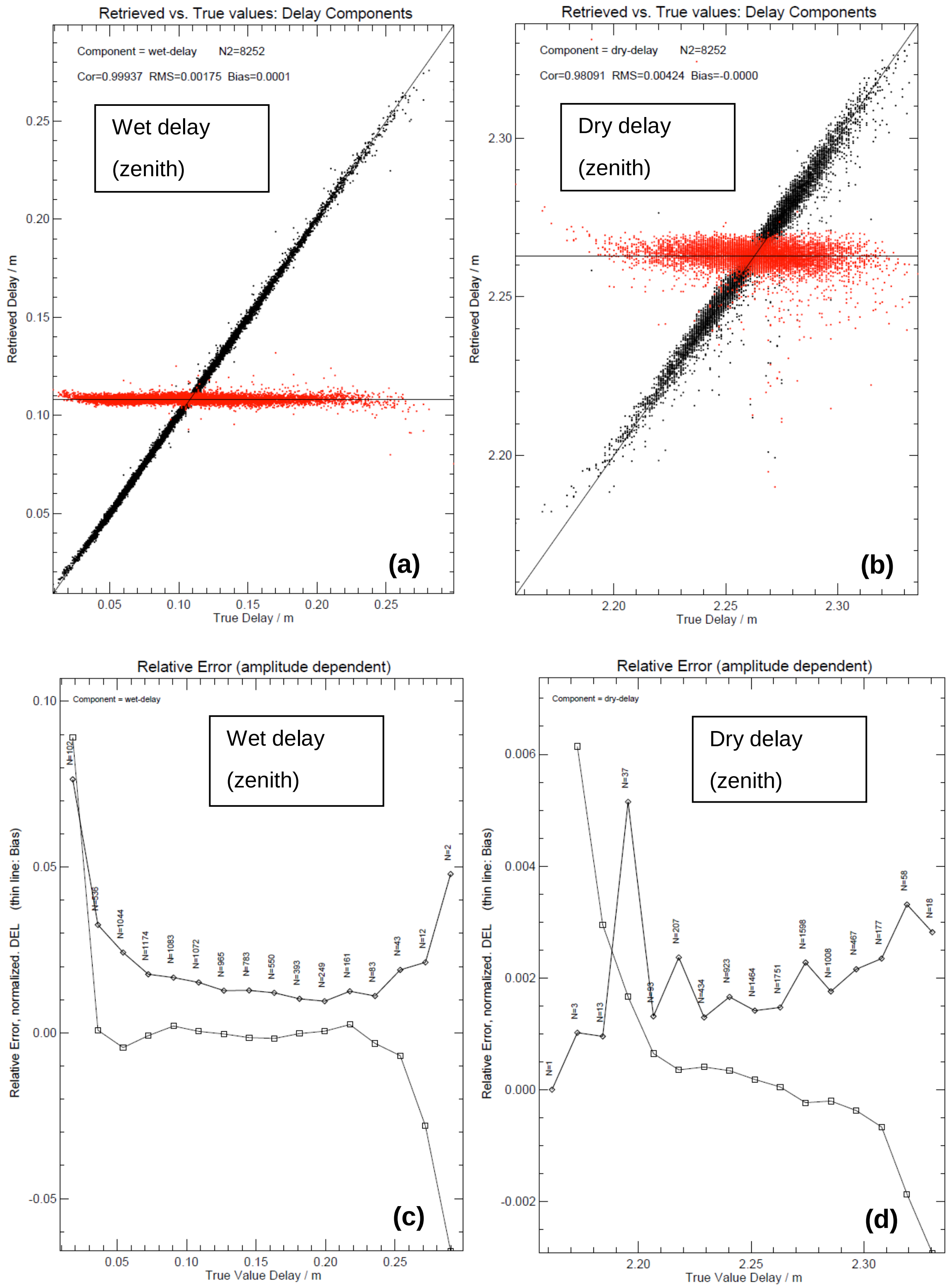

Figure 1 - Results of the self-consistency test of wet (a,c) and dry delays (b,d) for HATPRO zenith retrievals for RPG Meckenheim

Table 2 – Results of the self-consistency test of wet and dry delays for zenith

Parameter	Correlation coefficient	RMSE, mm	Relative error, %
Wet delay	0.999	1.75	~2 % (5…25 cm) ~9 % (< 5 cm)
Dry delay	0.981	4.24	~0.5 %

3 NN retrieval file structure and application

To apply the NN retrievals and derive dry and wet delays the following input parameters for the retrieval file *.RET are required:

a state vector X which includes a set of brightness temperature TBf1, TBf2 …TBf14, surface pressure PS, and cos(2*pi*DOY/365), sin(2pi*DOY/365), where DOY is a day of the year.

The following procedure should be applied to retrieve a parameter based on the provided retrieval files:

1) Input scaling

   Xn = (X - X_bias) ⋅ Xscale ,

   where X_bias is a 1st line of NS in the *.RET file and corresponds to ‘input_offset’, X_scale is a 2nd line of NS in the *.RET file (‘input_scale’)

2) Xn = [Xn 1] extend vector Xn by adding one more element at the end to account for a bias
3) n = Np ⋅ W1 ⋅ Xn, where Np is the smoothness variable for transfer function, W1 are NN coefficients or weights of the NN from *.RET file
4) n = tanh(n)
5) n = [n 1] extend vector n by adding one more element at the end to account for a bias
6) y = Np ⋅ W2 ⋅ n , where W2 are weights of the NN from *.RET file
7) y = tanh(y)
8) Output scaling

   y = y ⋅ Y_scale + Y_bias ,

   where y is the retrieved parameter, Y_bias is a 3rd line of NS (‘output_offset’), Y_scale is a 4th line of NS (‘output_scale’)

References

[1] J. M. Rüger, ”Refractive Index Formulae for Radio Waves”, Proceedings, FIG XXII International Congress, Washington DC, 19-26 April 2002. (https://www.fig.net/resources/proceedings/fig_proceedings/fig_2002/Js28/JS28_rueger.pdf)

[2] A. Martellucci and R. Prieto Cerdeira, "Review of tropospheric, ionospheric and multipath data and models for global navigation satellite systems," 2009 3rd European Conference on Antennas and Propagation, Berlin, Germany, 2009, pp. 3697-3702.