

Characterizing Atmospheric Turbulence and Instrumental Noise Using Two Simultaneously Operating Microwave Radiometers

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Abstract—We present an investigation on the correlation between slant wet delays in different directions using two radiometers. A scaling factor for the atmospheric turbulence is estimated on a daily basis using data from one single or both radiometers. The result agree at the $\sim 30\%$ level. We also make tests by increasing the integration in order to decrease the radiometer noise. We show that the retrieved atmospheric variability does not significantly depend on the integration time.

I. INTRODUCTION

Atmospheric variability affects the travel time of radio signals in the atmosphere. Such variability can be described by models based on theory of atmospheric turbulence [1]. Good modeling of atmospheric turbulence is important in e.g. applications where high accuracy determination of the propagation delay of radio signals are needed, for example in GPS meteorology [2]. In such applications it is useful to know the correlations between the wet atmospheric delays in different directions.

The wet atmospheric delay of radio signals of earth-satellite links can be inferred from ground-based microwave radiometry [3]. Hence it is possible to use microwave radiometers to test models. In [4] we tested a model describing the correlations between slant wet delays in different directions using the Astrid radiometer at the Onsala Space Observatory. In that work we had to assume that the atmosphere did not change significantly during a short time period (300 s) since we only used one radiometer and could hence only measure in one direction at the time. In this work we present results using data from two co-located radiometers (see Fig. 1 and Fig. 2). This gives us a variety of different combinations of data from the radiometers in order to derive parameters for atmospheric turbulence and instrumental noise. We also carry out an investigation on the impact of reducing the noise by increasing the integration time of the radiometer.

II. THEORY

The equivalent zenith wet delay (slant wet delay mapped to the zenith direction) of a radio signal observed in the direction

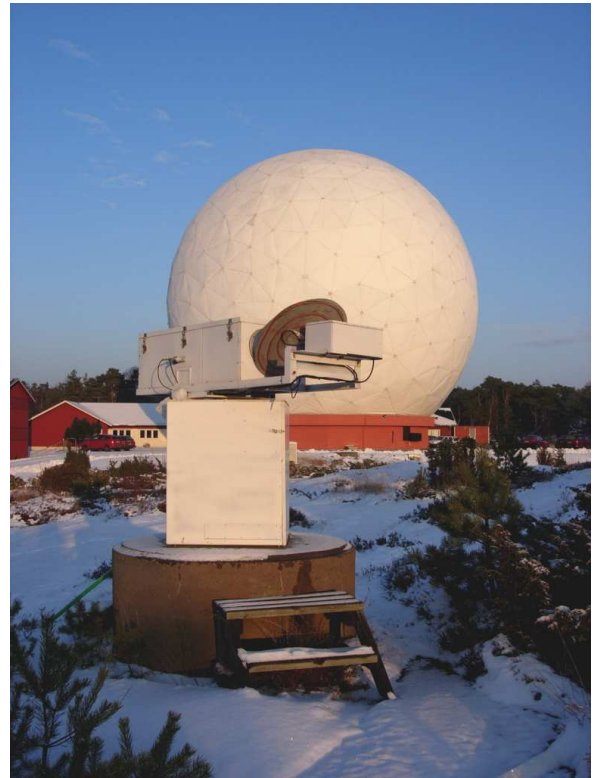


Fig. 1. The Astrid radiometer at the Onsala Space Observatory.

i is defined as:

$$\begin{aligned} l_i &= \frac{1}{m(\epsilon_i)} \int_S 10^{-6} N_w(\mathbf{r}_i(z)) ds \\ &= 10^{-6} \int_0^\infty N_w(\mathbf{r}_i(z)) dz \end{aligned} \quad (1)$$

where $m(\epsilon_i)$ is the mapping function for the elevation angle ϵ_i , N_w is the wet refractivity ($N_w = 10^6(n_w - 1)$, n_w being the wet part of the refractive index), S is the slant path taken by the signal, and $\mathbf{r}_i(z)$ is the position of the signal at height



Fig. 2. The Konrad radiometer at the Onsala Space Observatory.

z . The correlation between two equivalent zenith wet delays of two different directions (i and j) is given by [5], [6]:

$$\begin{aligned} \langle (l_i - l_j)^2 \rangle &= \frac{1}{2} \iint \langle [N_w(\mathbf{r}_i(z)) - N_w(\mathbf{r}_j(z'))]^2 \rangle dz dz' \\ &\quad - \frac{1}{2} \iint \langle [N_w(\mathbf{r}_i(z)) - N_w(\mathbf{r}_i(z'))]^2 \rangle dz dz' \\ &\quad + \frac{1}{2} \iint \langle [N_w(\mathbf{r}_j(z)) - N_w(\mathbf{r}_i(z'))]^2 \rangle dz dz' \\ &\quad - \frac{1}{2} \iint \langle [N_w(\mathbf{r}_j(z)) - N_w(\mathbf{r}_j(z'))]^2 \rangle dz dz' \end{aligned} \quad (2)$$

where $\langle \dots \rangle$ denotes expectation value and $\mathbf{r}_i(z)$ is the position of the ray in the direction i at the height z . According to [5] the expectation value of the squared difference between the wet refractivity at two locations $\mathbf{r}_i(z)$ and $\mathbf{r}_j(z')$ is:

$$\begin{aligned} \langle [N_w(\mathbf{r}_i(z)) - N_w(\mathbf{r}_j(z'))]^2 \rangle \\ = 10^{-12} C_n^2 |\mathbf{r}_i(z) - \mathbf{r}_j(z')|^{2/3} \end{aligned} \quad (3)$$

where C_n is the refractivity structure constant. Assuming that C_n is constant up to an effective tropospheric height h and zero above the integrals in (2) can be computed [5].

When testing the model using data from a water vapor radiometer we must also consider the effect of the instrumental noise. Since we are using squared differences of equivalent zenith wet delays, the effect of the noise will not be averaged out by using many observed differences. Instead the effect of the noise must be modelled. If we use equivalent zenith wet delays observed by one radiometer, the expectation value of the squared difference between two observed delays (\tilde{l}_i and \tilde{l}_j) can be expressed as [4]:

$$\begin{aligned} \langle (\tilde{l}_i - \tilde{l}_j)^2 \rangle &= k^2 \cdot \langle (l_i - l_j)^2 \rangle \Big|_0 \\ &\quad + \left(\frac{1}{m(\epsilon_i)^2} + \frac{1}{m(\epsilon_j)^2} \right) \cdot Var[B] \end{aligned} \quad (4)$$

where $Var[B]$ is the variance of the radiometer noise, $\langle (l_i - l_j)^2 \rangle \Big|_0$ are the expectation value according to (2), and

k^2 is a constant given by:

$$k^2 = \frac{C_n^2 h^{8/3}}{C_{n0}^2 h_0^{8/3}} \quad (5)$$

This constant is needed since the values of C_n and h may deviate from the a priori values used in the calculation of (2), C_{n0} and h_0 (in this work we use the values from [5]: $C_{n0}=2.4 \cdot 10^{-7} \text{ m}^{-1/3}$ and $h_0=1 \text{ km}$). Since a single radiometer can only measure in one direction at the time, we must assume that the atmosphere does not change significantly during a short period in which the radiometer can make several measurements in different directions. In [4] it was found that under most circumstances this time period could be chosen to be around 300 s.

To avoid possible problems due to temporal variations in the atmospheric refractivity more than one radiometer need to be used. Using two radiometers their noise variances will in general be different and we can write:

$$\begin{aligned} \langle (\tilde{l}_i - \tilde{l}_j)^2 \rangle &= k^2 \cdot \langle (l_i - l_j)^2 \rangle \Big|_0 \\ &\quad + \frac{1}{m(\epsilon_i)^2} \cdot Var[B_1] + \frac{1}{m(\epsilon_j)^2} \cdot Var[B_2] \end{aligned} \quad (6)$$

where $Var[B_1]$ and $Var[B_2]$ are the variances of the noise in the two radiometers. This model will hold if there are no biases between the two radiometers in the equivalent zenith wet delay. This can however not be expected to be the case in reality. In this work we dealt with this problem by estimating a slowly time varying bias between the radiometers before making the fit to model (6). The observed bias was smoothed and modeled as a piecewise linear function in 30 minutes intervals. When estimating k^2 and $Var[B]$ using (6) we used measurements from two radiometers that were acquired less than five seconds apart (we had to allow for the measurements to be a few seconds apart since the radiometers were not synchronized to measure at the same time).

For both models we estimate one k^2 value and one value for the noise variances for each one day period. We need a period of at least this length in order to estimate these parameters with good accuracy [4].

III. SIMULATIONS

We assessed the accuracy of the retrieval method based on the two models in a number of simulations. We first simulated slant wet delays behaving according to the model (2) (using the explicit equations (A8)–(A10) in [6]). As input to the simulations we used a zenith wet delay value (10 cm in all simulations) and a value of k^2 . One atmosphere (given by a set of slant wet delays) was simulated every 4 hours and the atmosphere in between was described as a linear combination of these. This corresponds to a slowly varying atmosphere, which can be seen as a constant flow of the air with a velocity of 1 m/s. The simulated period was one day. These atmospheres were then used to obtain the simulated radiometer observations. To these we added noise, and processed them using our normal radiometer retrieval algorithm to calculate

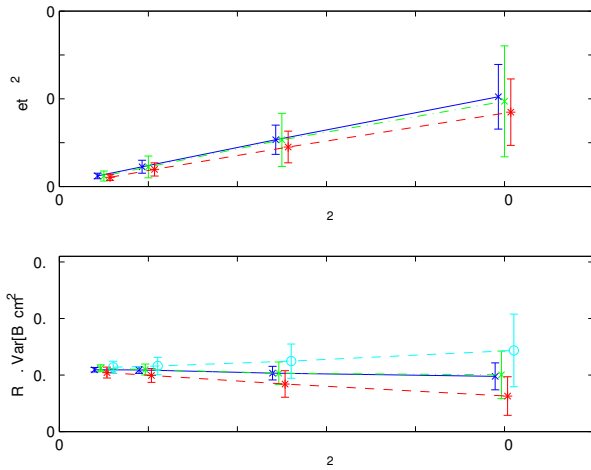


Fig. 3. Average retrieved values of k^2 and $Var[B]$ as function of the value of k^2 used in the simulations. The errorbar show the standard deviation. The result from using one radiometer is shown with a blue solid line (sky-mapping radiometer) and the green dashed-dotted line (elevation angle scanning radiometer). The combined results for k^2 are shown with the red dashed line in the upper plot and the with the red and cyan dashed lines in the lower plot.

the slant wet delays. These could then be used to estimate k^2 and the noise variances of the radiometers. By doing this, instead of using the simulated slant wet delays directly, we were able also to study errors introduced by the radiometer retrieval algorithm. We simulated two radiometers; one operating in a sky-mapping mode and one making elevation angle scans (between 20° and 160°) with the azimuth angle fixed.

First we made simulations to test the retrieval of k^2 and the noise variances using (6). We compared the result obtained with and without estimating a time-varying bias between the radiometers before the fitting to (6). The simulated k^2 value was 2 and the noise was set to zero. The average k^2 values from 100 simulation were 1.72 and 2.35 with and without the bias estimation. The standard deviation of the k^2 estimates was larger when we did not estimate the bias (0.91 compared to 0.74). This indicate that the bias estimation removes some atmospheric variability from the data, while we introduced errors by not doing it. It should be noted that the bias between the radiometers are likely to be larger with two real radiometers since the simulated radiometers were considered to be of the same type. For example, both radiometers were assumed to have pencil beam shaped antenna patterns. Furthermore, we do not include the effect of rain in the simulations which is an important error source. The bias in the simulations comes from different errors in tip-curve calibration of the two radiometers. With two real radiometers we are likely to also have other effects, for example if the radiometers are operating at different frequencies this may also contribute to a bias.

In another set of simulations the variance of the noise was 0.25 K^2 (approximately corresponding to 0.10 cm^2 in

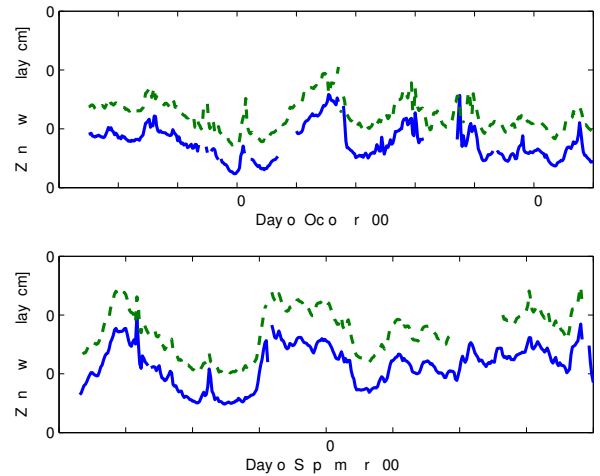


Fig. 4. Time-series of the zenith wet delay from Astrid (blue solid line) and Konrad (green dashed line) during CONT02 (upper plot) and CONT05 (bottom plot). The Konrad data are offset by 5 cm for visibility.

the wet delay). Fig. 3 shows the retrieved values of k^2 and $Var[B]$ from these as function of the value of k^2 used in the simulations. Each point displays the average obtained value of 100 simulations with an errorbar representing the standard deviation. As seen the retrieved values of k^2 on average agrees with those used in the simulations when using the model (4), while the model (6) on average slightly underestimates k^2 . We can also see that the uncertainty of the estimation using (4) is larger for the second radiometer which only scans in elevation, which is something that can be expected since the first radiometer scans the whole sky.

The reason for k^2 not being estimated better is that the simulated atmosphere was not varying much in time, hence there was not enough information about this parameter available in the simulated observations. We only simulated a new atmosphere every 4 hours, hence we will have observations of only 6 independent atmospheres per day. Using a more variable simulated atmosphere where a new atmosphere is instead updated every hour, the standard deviations of the retrieved daily k^2 values is reduced by about 50%. This can be expected because the daily value is then inferred from 24, rather than 6, noisy atmospheres. We also investigated the impact of varying the radiometer noise and found that the noise had no significant impact on the error in k^2 , showing that it is possible to separate the atmospheric variability and the atmospheric noise.

IV. RESULTS

A. CONT experiments

During the two continuous VLBI (Very Long Baseline Interferometry) experiments CONT02 (15–31 October 2002) and CONT05 (12–27 September 2005) there were two water vapor radiometers operating at the Onsala Space Observatory: the Astrid radiometer [7] (see Fig.1) and the Konrad radiometer

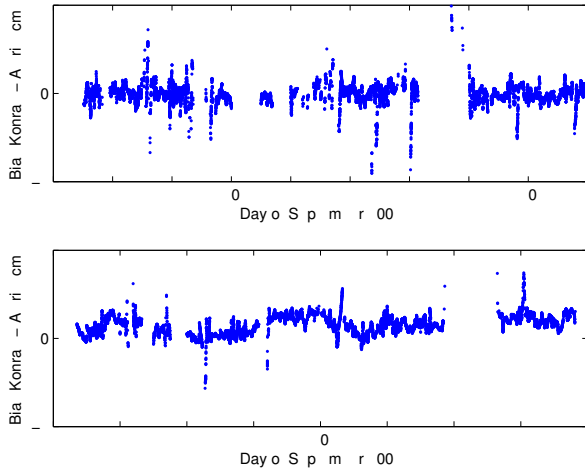


Fig. 5. Estimated zenith wet delay bias between the two radiometers from CONT02 and CONT05.

[8] (see Fig.2). These periods provided data to test the models (4) and (6). One test we did was to apply the model (4) to each of the radiometers to see if they gave consistent results. We also used the model (6) and compare the result to that of (4). In CONT02 the Astrid radiometer was operating in a continuous sky-mapping mode and in CONT05 it did elevation angle scans between 20° and 160° elevation angle (azimuth angle fixed to the north direction). In both experiments the Konrad radiometer was slaved to follow the VLBI schedule. Fig. 4 shows time-series equivalent zenith wet delay inferred from the two radiometers during these two periods.

In Fig. 5 the estimated zenith wet delay bias between the two radiometers is shown. The average bias for CONT02 was 0.4 mm and for CONT05 it was 7.5 mm. The bias was estimated as a piece-wise linear function in 30 min intervals. We also tested using other intervals, but 30 min were found to give the best results when estimating k^2 and the instrumental noise variances.

In Fig. 6 the retrieved values for k^2 and the noise variances during CONT02 are shown, and Fig. 7 shows the corresponding for CONT05. For the CONT02 period there were no useful data for the October 27 due to rain. During CONT05 we had problems with the pointing on September 24, hence we had no Konrad data for this day. As seen the results agree rather well. The average values for k^2 and the noise variances can be seen in Table I.

The noise level for the Konrad radiometer was lower than that for Astrid. One reason is that the integration times for Konrad is larger: ~ 10 s compared to ~ 1 s for Astrid. The noise for Astrid was significantly lower in CONT05 than in CONT02, a result of an upgrade of the Astrid data acquisition system in the beginning of 2003.

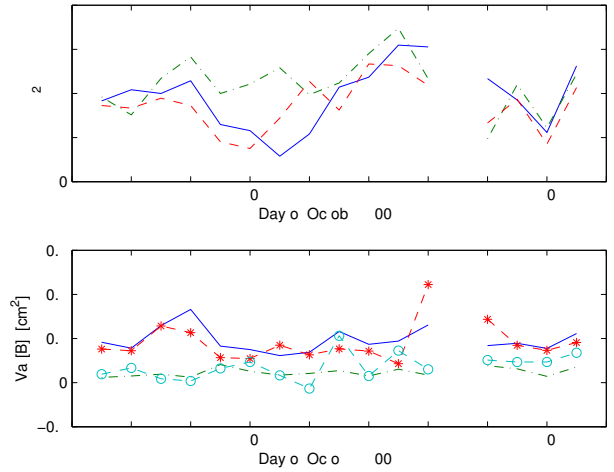


Fig. 6. Retrieved values of k^2 and $Var[B]$ for the CONT02 period. The results using (4) are the blue solid line (Astrid) and the green dash-dotted line (Konrad). The results using (6) are the dashed lines. For the noise estimates in the lower plot the red line with asterixes denotes Astrid data and the cyan line with rings denotes Konrad data.

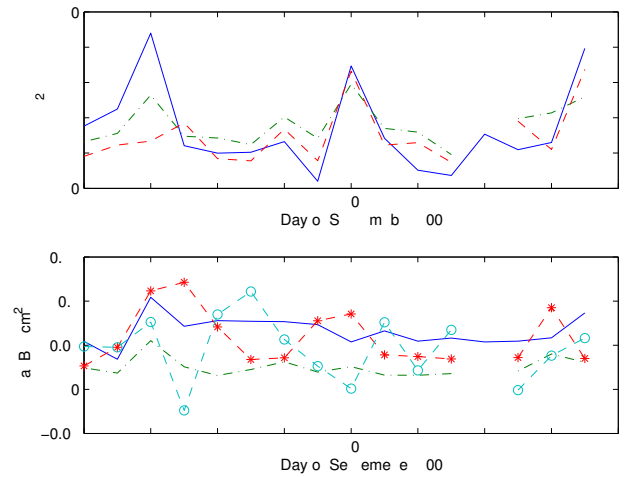


Fig. 7. Retrieved values of k^2 and $Var[B]$ for the CONT05 period. See caption of Fig. 6.

B. Integration time test

We studied on which level the integration time affects the result. The Astrid radiometer was operating in a sky-mapping schedule, making eight consecutive measurements in each direction separated by 1.5 s. By using only one of these measurements or a mean of several of them, the integration time could be varied.

Fig. 8 shows the retrieved values of k^2 and $Var[B]$ for different integration times. Shown are the result when using one measurement, using the mean of two measurements (the first and the last, approximately 11 s apart), and using a mean of all eight measurements. We note that the retrieved values of

TABLE I
THE AVERAGE k^2 AND $Var[B]$ VALUES FROM CONT02 AND CONT05,
RETRIEVED USING THE TWO MODELS (4) AND (6).

Period	Radiometer	Model	Mean k^2	$Var[B]$ [cm^2]
CONT02	Astrid	(4)	1.9	0.096
	Konrad	(4)	2.2	0.023
	Both	(6)	1.7	
	Astrid	(6)		0.091
	Konrad	(6)		0.036
CONT05	Astrid	(4)	3.4	0.066
	Konrad	(4)	3.6	0.025
	Both	(6)	3.0	
	Astrid	(6)		0.059
	Konrad	(6)		0.046

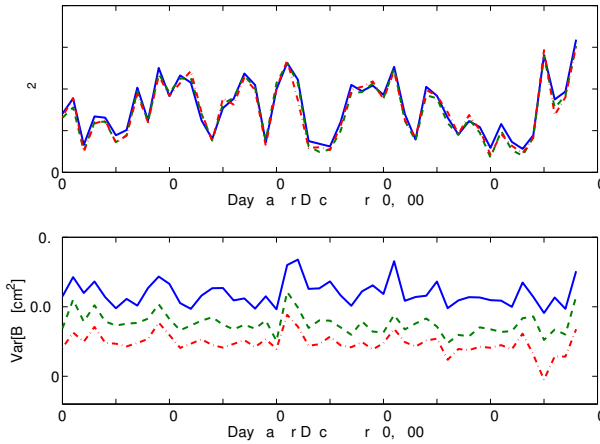


Fig. 8. The retrieved values of k^2 and $Var[B]$ for the period 20 December 2005 – 6 February 2006, using Astrid with different integration times accomplished by taking the mean of several measurements in the same direction. Shown are the result using one measurement (blue solid line), mean of two measurements (green dashed line), and mean of eight measurements (red dashed-dotted line).

k^2 are insensitive to the integration time. The RMS difference between the k^2 values retrieved using two measurements and those retrieved using one measurement was 0.12. The RMS difference between k^2 using eight and using one measurement was 0.15, and between using two and eight measurements it was 0.11.

Theoretically, if the radiometer noise of all measurements was uncorrelated, the noise variance would be inversely proportional to the integration time. The retrieved values of $Var[B]$ are on average 0.060 cm^2 , 0.038 cm^2 , and 0.024 cm^2 using averages of one, two, and eight measurements respectively, hence this is not the case. One explanation for this could be that some atmospheric variability is erroneously interpreted as radiometer noise. If this would be the case we would expect that there is a correlation between the retrieved noise level and the retrieved value of k^2 . We tested this by comparing the noise variances retrieved from days with $k^2 > 1.5$ (23 days) with those from days with $k^2 < 1.5$ (26 days). For

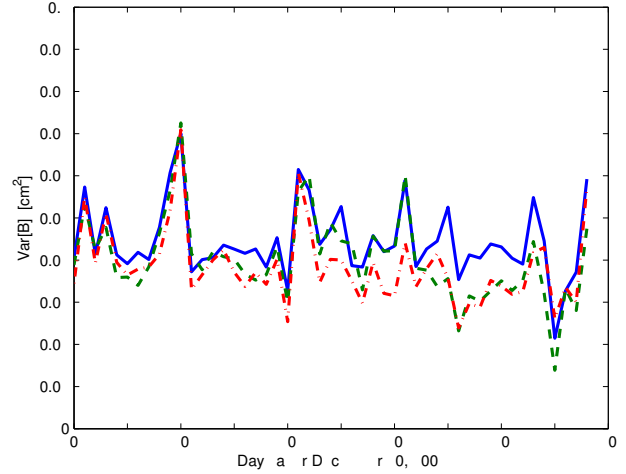


Fig. 9. The retrieved values of $Var[B]$ for the period 20 December 2005 – 6 February 2006, using Astrid when taking the mean of two measurements in the same direction. Shown are the result using two measurement separated by $\sim 1.5\text{ s}$ (blue solid line), $\sim 6\text{ s}$ (green dashed line), and $\sim 11\text{ s}$ (red dashed-dotted line).

the period with high k^2 values the average $Var[B]$ values were 0.061 cm^2 , 0.039 cm^2 , and 0.025 cm^2 , using averages of one, two and eight measurements respectively. For the low k^2 period the corresponding values were 0.058 cm^2 , 0.037 cm^2 , and 0.023 cm^2 . Hence the noise variance is $0.002\text{--}0.003\text{ cm}^2$ larger for the high k^2 days. This indicates that there might be some atmospheric variability interpreted as noise, although a longer period should be investigated to draw any definite conclusions. The difference is also rather small, hence this cannot entirely explain the noise not being inversely proportional to the integration time.

A more likely explanation is that the noise of the measurements was correlated. Some correlations in the noise can be expected over short timescales since Astrid uses hardware integration (first order RC-circuit) with time-constant 1 s. Hence, combining measurements 1.5 s apart the noise level will be reduced by $\sim 40\%$ compared to that of a single measurement, rather than the 50% expected for a perfect white noise process. Over longer timescales like 10 s there is no significant correlation.

This can be tested by using e.g. the mean of two measurements in the retrieval of k^2 and $Var[B]$ in model (4), and varying which two measurements that are used. If the noise is correlated, the correlation can be expected to decrease with increasing time between the two used measurements. When using the mean of the first and second measurements the retrieved value of $Var[B]$ was on average 0.044 cm^2 , when using the first and the fifth it was 0.040 cm^2 , and when using the first and the seventh it was 0.039 cm^2 . The time series of these estimated variances are plotted in Fig. 9. The results indicate that there are correlations in the radiometer noise.

V. CONCLUSIONS

The results from the two CONT experiments agree rather well in general. The observed difference between the retrieved values of k^2 is at the level which can be expected from the simulation results. The agreement between the k^2 values retrieved using one radiometer in model (4) as in [4] and using two radiometers and model (6) indicates that the method using only one radiometer and assuming no significant variations in time on timescales <300 s works well.

As seen the noise variances retrieved using (4) were rather constant during respective period. The noise for Astrid was lower during CONT05 than during CONT02, which was expected due to an upgrade of the radiometer in 2003. This agrees well with the result in [4]. The noise variances retrieved using (6) varies much more, especially those from CONT05. This can be expected from the simulation results which show that the uncertainty of the noise retrieved using this model are larger than the noise retrieved using (4), especially when k^2 is large (note that k^2 is larger in CONT05 than in CONT02). There are some days where the results are not consistent. One reason for the disagreement on 20–21 October 2002 may be that much of the data on this day were not usable due to rain. Hence the results for these days are based on less data than for other days and can be expected to be more uncertain. Another explanation is that Konrad was slaved to the VLBI schedule and this may not be an optimum schedule to obtain data to be used to investigate correlations between slant wet delays in different directions. In CONT05 the Astrid did only do elevation angle scans, hence did not map the whole sky as in CONT02. This is likely to have degraded the results and may be one explanation for the noise during CONT05 retrieved by model (6) being more variable. Instrumental differences between the two radiometers are likely impact the results. For example the two radiometers have different beamwidths (6°

for Astrid and 3° for Konrad) and this can be expected to affect the result at some level.

The investigation regarding the integration time shows that the value of k^2 estimated using (4) is relatively independent of the integration time. Hence an increase of the integration time to at least ~ 10 s (and hence decrease the noise) will not lead to any significant loss of information about the atmosphere in terms of the model parameters estimated in this work.

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