Characterizing Atmospheric Turbulence and Instrumental Noise Using Two Simultaneously Operating Microwave Radiometers

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***Abstract*— We present an investigation on the correlation between slant wet delays in different directions using two radiometers. A scaling factor for the atmospheric turbulence is estimated using data from one single or both radiometers. The result agree at the $\sim$30% level. We also make tests by increasing the integration in order to decrease the radiometer noise. We show that the retrieved atmospheric variability does not significantly depend on the integration time.**

I. Introduction

Atmospheric variability affects the travel time of radio signals in the atmosphere. Such variability can be described by models based on theory of atmospheric turbulence [1]. Good modeling of atmospheric turbulence is important in e.g. applications where high accuracy determination of the travel time of radio signal are needed, for example in GPS meterology [2]. In such applications it is useful to know e.g. the correlations between the wet atmospheric delays in differnet directions.

The wet atmospheric delay of radio signals of earth satellite links can be inferred from ground-based microwave radiometry [3]. Hence it is possible to use microwave radiometers to test models. In [4] we tested a model describing the correlations between slant wet delays in different directions using the Astrid radiometer at the Onsala Space Observatory. In that work we had to assume that the atmosphere did not change significantly during a short time period (300 s) since we only used one radiometer and could hence only measure in one direction at the time. In this work we present results using data from two co-located radiometers (see Fig. 1 and Fig. 2). This gives us a variety of different combinations of data from the radiometers in order to derive parameters for atmospheric turbulence and instrumental noise. We also make investigation on the impact of reducing the noise by increasing the integration time of the radiometer.

II. Theory

The equivalent zenith wet delay (slant wet delays mapped to zenith) of a radio signal observed in the direction i, l_i is defined as:

$$l_i = \frac{1}{m(\epsilon_i)} \int_S 10^{-6} N_w(\mathbf{r}_i(z)) ds = 10^{-6} \int_0^\infty N_w(\mathbf{r}_i(z)) dz \quad (1)$$

where $m(\epsilon_i)$ is the mapping function for elevation angle ϵ_i, N_w the wet refractivity ($N_w = 10^6(n_w - 1)$, n_w being the wet part of the refractive index), S the slant path taken by the signal, and $\mathbf{r}_i(z)$ the position of the signal at height z. The correlation between two equivalent zenith wet delays of two

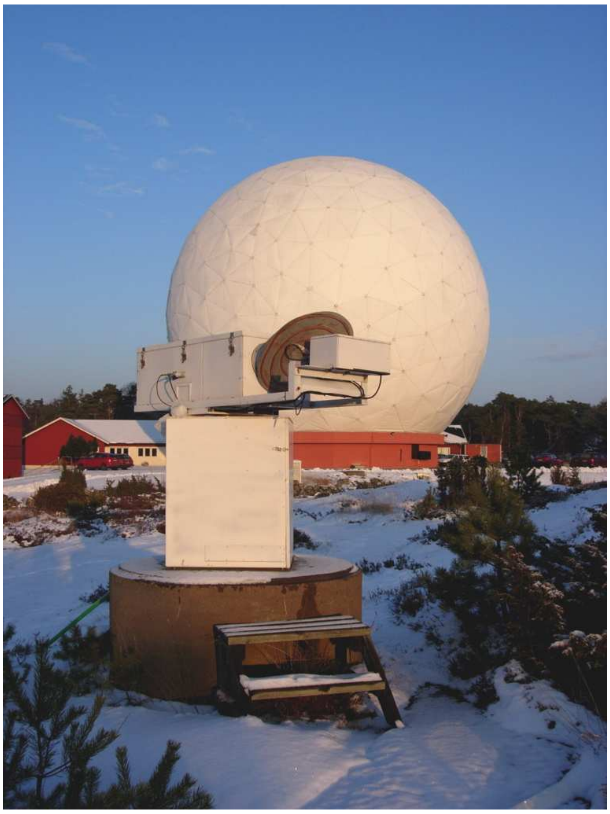

Fig. 1. The Astrid radiometer at the Onsala Space Observatory.

Fig. 2. The Konrad radiometer at the Onsala Space Observatory.

differnet directions (i and j) is given by [5], [6]:

$$\begin{aligned}\left\langle (l_i - l_j)^2 \right\rangle = & \frac{1}{2} \iint \left\langle [N_w(\mathbf{r}_i(z)) - N_w(\mathbf{r}_j(z'))]^2 \right\rangle dz dz' \\ & -\frac{1}{2} \iint \left\langle [N_w(\mathbf{r}_i(z)) - N_w(\mathbf{r}_i(z'))]^2 \right\rangle dz dz' \\ & +\frac{1}{2} \iint \left\langle [N_w(\mathbf{r}_j(z)) - N_w(\mathbf{r}_i(z'))]^2 \right\rangle dz dz' \\ & -\frac{1}{2} \iint \left\langle [N_w(\mathbf{r}_j(z)) - N_w(\mathbf{r}_j(z'))]^2 \right\rangle dz dz' \end{aligned} \quad (2)$$

where $\langle \ldots \rangle$ denotes expectation value and $\mathbf{r}_i(z)$ is the position of the ray in the direction i at the height z. According to [5] the expectation value of the squared difference between the wet refractivity at two locations $\mathbf{r}_i(z)$ and $\mathbf{r}_j(z')$ is:

$$\left\langle [N_w(\mathbf{r}_i(z)) - N_w(\mathbf{r}_j(z'))]^2 \right\rangle = 10^{-12} C_n^2 \left| \mathbf{r}_i(z) - \mathbf{r}_j(z') \right|^{2/3} \quad (3)$$

where C_n is the refractivity structure constant. Assuming that C_n is constant up to an effective tropospheric height h and zero above the integrals in (2) can be computed [5].

When testing the model using data from a water vapor radiometer we must also consider the effect of the instrumental noise. Since we are using squared differences of equivalent zenith wet delays, the effect of the noise will not be averaged out by using many observed differences. Instead the effect of the noise must be modelled. If we use equivalent zenith wet delays observed by one radiometer, the expectation value of the squared difference between two observed delays ($\tilde{l}_i$ and $\tilde{l}_j$) can be expressed as [4]:

$$\begin{aligned}\left\langle \left(\tilde{l}_i - \tilde{l}_j\right)^2 \right\rangle = & \; k^2 \cdot \left\langle (l_i - l_j)^2 \right\rangle \Big|_0 \\ & + \left(\frac{1}{m(\epsilon_i)^2} + \frac{1}{m(\epsilon_j)^2} \right) \cdot Var[B] \end{aligned} \quad (4)$$

where $Var[B]$ is the variance of the radiometer noise, $\left\langle (l_i - l_j)^2 \right\rangle \Big|_0$ are the expection value according to (2), and k^2 is a constant given by:

$$k^2 = \frac{C_n^2 h^{8/3}}{C_{n0}^2 h_0^{8/3}} \quad (5)$$

This constant is needed since the values of C_n and h may deviate from the a priori values used in the calculation of (2), C_{n0} and h_0 (in this work we use the values from [5]: C_{n0} =2.4 m$^{-1/3}$ and h_0=1 km). Since a single radiometer can only measure in one direction at the time, we must assume that the atmosphere does not change significantly during a short period in which the radiometer can make several measurements in different directions. In [4] it was found that under most circumstances this time period could be chosen to be around 300 s.

To avoid possible problems due to temporal variations in the atmospheric refractivity more than one radiometer need to be used. Using two radiometers their noise variances will in general be different. Hence in this case we will have:

$$\begin{aligned}\left\langle \left(\tilde{l}_i - \tilde{l}_j\right)^2 \right\rangle = & \; k^2 \cdot \left\langle (l_i - l_j)^2 \right\rangle \Big|_0 \\ & + \frac{1}{m(\epsilon_i)^2} \cdot Var[B_1] + \frac{1}{m(\epsilon_j)^2} \cdot Var[B_2] \end{aligned} \quad (6)$$

where $Var[B_1]$ and $Var[B_2]$ are the variances of the noise in the two radiometers. This model will hold if there are no biases between the two radiometers in the equivalent zenith wet delay. This can however not be expected to be the case in reality. In this work we dealt with this problem by estimating a slowly time varying bias between the radiometers before making the fit to model (6). The observed bias was smoothed and modeled as a piecewise linear function in 30 min intervals. When estimating k^2 and $Var[B]$ using (6) we used measurements from two radiometers that were acquired less than five secons apart (we had to allow for the measurements to be a few seconds apart since the radiometers were not synchronized to measure at the same time).

For both models we estimate one k^2 value and one value for the noise variances for one day period. We need a period of at least this length in order to estimated these parameters with good accuracy [4].

III. SIMULATIONS

We assessed the accuracy of the retrieval method based on the two models in a number of simulations. We first simulated slant wet delays behaving according to the model (2) using the explicit equations (A8)–(A10) in [6]. As input to the simulations we used a zenith wet delay value (we used 10 cm in all simulations) and a value of k^2. One atmosphere (given by a set of slant wet delays) were simulated every 4 hours and the atmosphere in between was described as a linear combination of these. This corresponding to a slowly varying atmosphere, somewhat corresponding to a constnat flow of the air with a velocity 1 m/s. The simulated period was one day. These atmospheres were then used to obtain the simulated radiometer observations. To these we added noise, and processed them using our normal radiometer retrieval algorithm to retrieve

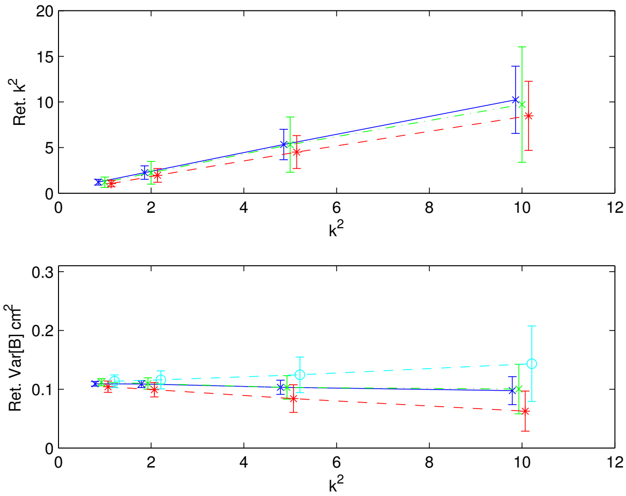

Fig. 3. Average retrieved values of k^2 and $Var[B]$ as function of the value of k^2 used in the simulations. The errorbar show the standard deviation. The result from using one radiometer is shown with a blue solid line (sky-mapping radiometer) and the green dashed-dotted line (elevation angle scanning radiometer). The combined results are shown with the dashed lines.

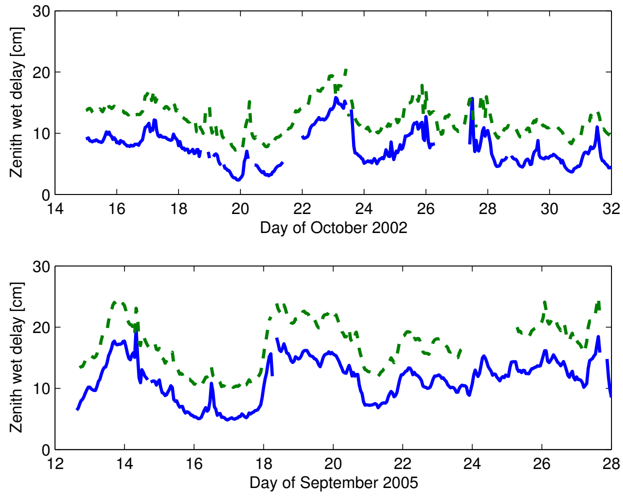

Fig. 4. Time-series of the zenith wet delay from Astrid (blue solid line) and Konrad (green dashed line) during CONT02 (upper plot) and CONT05 (bottom plot). The Konrad data are offset by 5 cm.

the slant wet delays. These could then be used to retrieve k^2 and the noise variances. By doing this, instead of using the simulated slant wet delays directly, we were able also to study errors introduced by the radiometer retrieval algorithm. We simulated two radiometers; one operating in a sky-mapping mode and one radiometer making elevation angle scans (between 20° and 160°) with the azimuth angle fixed.

First we made simulations to test the retrival of k^2 and the noise variances using (6). We compared the result obtained with and without estimating a time-varying bias between the radiometers before the fitting to (6). The simulated k^2 value was 2 and the noise was set to zero. The average k^2 values from 100 simulation were 2.35 and 1.72 when without and with the bias estimation. The standrad deviation of the k^2 estimates was larger when we did not estimate the bias (0.91 compared to 0.74). This indicate that the bias estimation removes some atmospheric variability from the data, while we introduced errors by not doing it. It should be noted that the bias between the radiometers are likely to be larger with two real radiometers since the simulated radiometers were considered to be equal. The bias in the simulations comes from different errors in tip-curve calibration of the two radiometers. With two real radiometers we are likely to also have other effects, for example if the radiometers are operating at different frequencies this may also to contribute to a bias.

In another set of simulations the variance of the noise was 0.25 K^2 (approximately corresponding to 0.10 cm^2 in wet delay). Fig. 3 shows the retrieved values of k^2 and $Var[B]$ from these as function of the value of k^2 used in the simulations. Each point displays the average retrived value of 100 simulations with an errorbar representing the standard deviation. As seen the retrieved values of k^2 on average agrees with those used in the simulations when using the model (4), while the model (6) on average slightly underestimates k^2. We can also see that the uncertainty of the estimation using (4) is larger for the second radiometer which only scans in elevation, which is something that can be expected since the first radiometer scans the whole sky.

The reason for k^2 not being estimated better is that the simulated atmosphere was not varying much in time, hence there were not enough information about this parameter available in the simulated observations. We only simulated a new atmosphere every 4 hours, hence we will have observations of only 6 independent atmospheres per day. Using a more variable simulated atmosphere where a new atmosphere is updated every hour instead, the standrad deviations of the retrived daily k^2 values is reduced by about 50%. This can be expected because the daily value is then inferred from 24, rather than 6, noisy atmospheres. We also investigated the impact of varying the radiometer noise and found that the noise had no significant impact on the error in k^2, showing that it is possible to separate the atmospheric variability and the atmospheric noise.

IV. Results

A. CONT experiments

During the two continuous VLBI (Very Long Baseline Interferometry) experiments CONT02 (15–31 October 2002) and CONT05 (12–27 September 2005) there were two water vapor radiometers operating at the Onsala Space Observatory: the Astrid radiometer [7] (see Fig.1) and the Konrad radiometer [8] (see Fig.2). These periods provided data to test the models (4) and (6). One test we did was to apply the model (4) to each of the radiometers to see if they gave consistent results. We also used the model (6) and compare the result to that of (4). In CONT02 the Astrid radiometer was operating in a

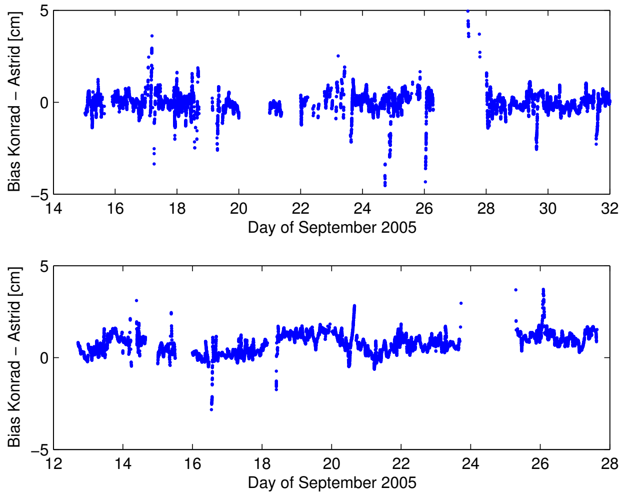

Fig. 5. Estimated zenith wet delay bias between the two radiometers from CONT02 and CONT05.

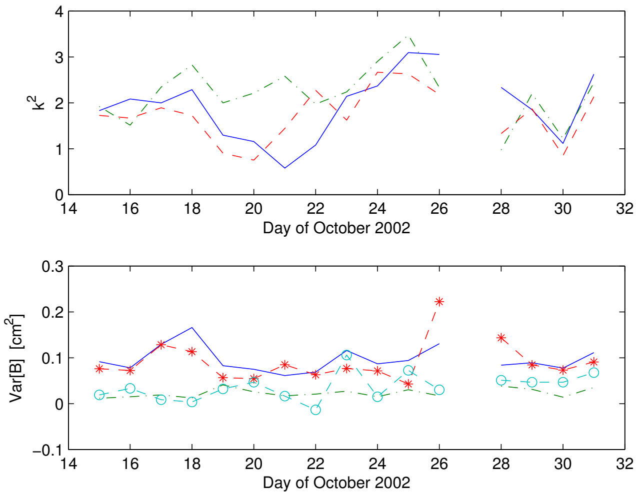

Fig. 6. Retrieved values of k^2 and $Var[B]$ for the CONT02 period. The results using (4) are the blue solid line (Astrid) and the green dash-dotted line (Konrad). The results using (6) are the dashed lines; for the noise the red line with asterixes is Astrid and cyan line with rings is Konrad.

continuous sky-mapping mode and in CONT05 it did elevation angle scans between 20° and 160° elevation angle (azimuth angle fixed). In both experiments the Konrad radiometer was slaved to follow the VLBI schedule. Fig. 4 shows time-series equivalent zenith wet delay inferred from the two radiometers during these two periods.

In Fig. 5 the estimated zenith wet delay bias between the two radiometers is shown. The average bias for CONT02 was 0.4 mm and for CONT05 it was 7.5 mm. The bias was estimated as a piece-wise linear function in 30 min intervals. We also tested using other lengths on the interval, but 30 min intervals were found to give the best results for when afterwards estimating k^2 and the instrumental noise variances.

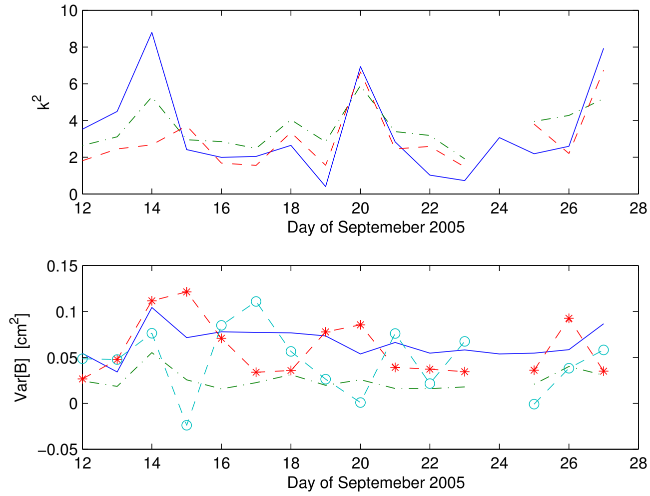

Fig. 7. Retrieved values of k^2 and $Var[B]$ for the CONT05 period. See caption of Fig. 6.

TABLE I

THE AVERAGE k^2 AND $Var[B]$ VALUES FROM CONT02 AND CONT05, RETRIEVED USING THE TWO MODELS (4) AND (6).

Period	Radiometer	Model	Mean k^2	$Var[B]$ [cm^2]
CONT02	Astrid	(4)	1.9	0.096
	Konrad	(4)	2.2	0.023
	Both	(6)	1.7	
	Astrid	(6)		0.091
	Konrad	(6)		0.036
CONT05	Astrid	(4)	3.4	0.066
	Konrad	(4)	3.6	0.025
	Both	(6)	3.0	
	Astrid	(6)		0.059
	Konrad	(6)		0.046

In Fig. 6 the retrieved values for k^2 and the noise variances during CONT02 are shown, and Fig. 7 shows the same for CONT05. For the CONT02 period there were no useful data for the October 27 due to rain. In the CONT05 period we hade problems with the pointing on September 24, hence we had no Konrad data this day. As seen the results agree rather well. The average values for k^2 and the noise variances can be seen in Table I.

As seen the noise level for the Konrad radiometer was lower than for Astrid. One reason is that the integration times for Konrad is larger: ∼10 s compared to ∼1 s for Astrid. The noise for Astrid was significantly lower in CONT05 than in CONT02, a result of an upgrade of the Astrid data acquisition system in beginning of 2003.

B. Integration time test

We studied on which level the integration time affects the result. The Astrid radiometer was operating in a sky-mapping schedule, making eight consecutive measurements in each direction separated by 1.5 s. By using only one of this measurements or a mean of several of them the integration time could then be varied.

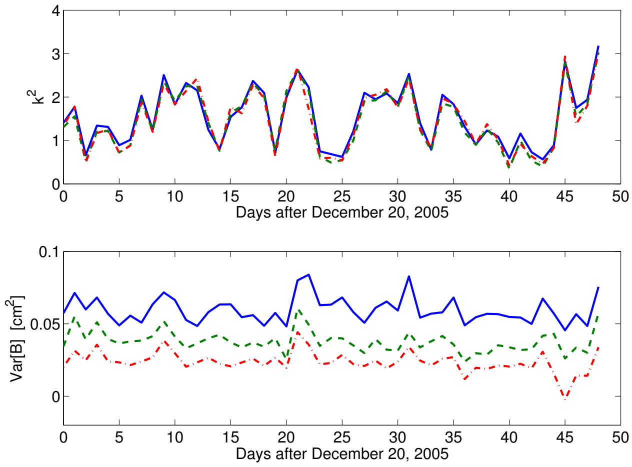

Fig. 8. The retrieved values of k^2 and $Var[B]$ for the period 20 December 2005 – 6 February 2006, using Astrid with different integration times accomplished by taking the mean of several measurements in the same direction. Shown are the result using one measurement (blue solid line), mean of two measurements (green dashed line), and mean of eight measurements (red dashed-dotted line).

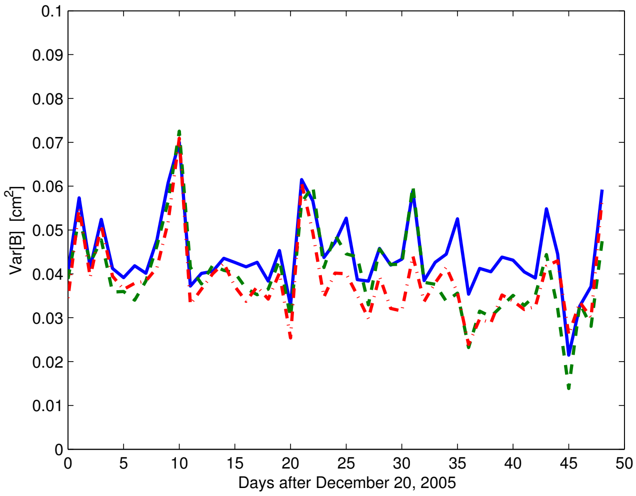

Fig. 9. The retrieved values of $Var[B]$ for the period 20 December 2005 – 6 February 2006, using Astrid when taking the mean of two measurements in the same direction. Shown are the result using two measurement separated by ∼1.5 s (blue solid line), ∼6 s (green dashed line), and ∼11 s (red dashed-dotted line).

Fig. 8 shows the retrived values of k^2 and $Var[B]$ for different integration times. Shown are the result when using one measurement, using the mean of two measurements (the first and the last, approximately 11 s apart), and using a mean of all eight measurements. We can see that the retrieved values of k^2 are insensitive to the integration time. The RMS difference between the k^2 values retrieved using two measurements and those retrived using one measurement was 0.12. The RMS difference between k^2 using eight and using one measurement was 0.15, and between using two and eight measurements it was 0.11.

Theoretically, if the radiometer noise of all measurements were uncorrelated, the noise would be inversely proportional to the integration time. The retrived values of $Var[B]$ are on average 0.060 cm^2, 0.038 cm^2, and 0.024 cm^2 using averages of one, two, and eight measurements respectively, hence this is not the case.

One explanation for this not being so could be that some atmospheric variability is erroneously interpreted as radiometer noise. If this would be the case we would expect that there is a correlation between the retrieved noise level and the retrieved value of k^2. We tested this by comparing the noise variances retrieved from days with $k^2 > 1.5$ (23 days) with those from days with $k^2 < 1.5$ (26 days). For the period with high k^2 values the average $Var[B]$ values were 0.061 cm^2, 0.039 cm^2, and 0.025 cm^2, using averages of one, two and eight measurements respectively. For the low k^2 period the corresponding values were 0.058 cm^2, 0.037 cm^2, and 0.023 cm^2. Hence the noise variance is 0.002–0.003 cm^2 larger for the high k^2 days. This indicate that there might be some atmospheric variability interpreted as noise, although a longer period should be investigated to draw any definite conclusions. The difference is also rather small, hence this cannot entirely explain the noise not being inversely proportional to the integration time.

The likely explanation for the retrived noise not being inversely proportional to the integration time is that the noise of the measurements were correlated. Some correlations in the noise can be expected over short timescales since Astrid uses hardware integration (first order RC-circuit) with time-constant 1 s. Hence, combining measurements 1.5 s apart the noise level will be reduced by ∼40% compared to that of a single measurement, rather than the 50/

This can be tested by using e.g. the mean of two measurements in the retrival of k^2 and $Var[B]$, and varying which two measurements that are used. If the noise is correlated, the correlation can be expected to decrease with increasing time between the two used measurements. When using the mean of the first and second measurements the retrieved value of $Var[B]$ was on average 0.044 cm^2, when using the first and the fifth it was 0.040 cm^2, and when using the first and the seventh it was 0.039 cm^2. The time series of these estimated variances are plotted in Fig. 9. The results indicate that there are correlations in the radiometer noise.

V. Conclusions

The results from the two CONT experiments agree rather well in general, the observed difference between the retrived values of k^2 is at the level which can be expected from the simulation results. The agreement between the k^2 values retrieved using one radiometer (4) as in [4] and using two radiometers and (6) indicates the method using only one radiometer and assuming no significant variations in time on timescales <300 s works well.

As seen the noise variances retrived using (4) were rather constant during respective period. The noise for Astrid was lower during CONT05 than during CONT02, which was expected due to an upgrade of the radiometer in 2003. This agrees well with the result in [4] which also shows a reduction of the noise as result of the upgrade. The noise variances retrieved using (6) varies much more, especially those from CONT05. This can be expected from the simulation results which shows that the uncertainty of the noise retrieved using this model are larger than the noise retrieved using (4), especially when k^2 is large (note that k^2 is larger in CONT05 than in CONT02).

There are some days where the results are not consistent. One reason for the disagreement on 20–21 October 2002 may be that much of the data on this day were not suable due to e.g. rain, hence the results for these days are based on less data than for other days and the results can be expected to be more uncertain. Another explanation is that Konrad was slaved to the VLBI schedule and this may not be an optimum schedule to obtain data to be used to investigate correlations between slant wet delays in different directions. In CONT05 the Astrid did only do elevation angle scans, hence did not map the whole sky as in CONT02. This is likely to have degraded the results and may be one explanation for the noise during CONT05 retrived by model (6) being very variable. Instrumental differences between the two radiometers are likely to have had an impact on the results. For example the two radiometers have different beamwidths (6° for Astrid and 3° for Konrad) and this can be exected to affect the result on some level.

The investigation regarding the integration time shows that the value of k^2 estimated using (4) is relatively independent of the integration time. Hence it would seem that we can increase the integration time to at least $\sim$10 s (and hence decrease the noise) without any significant loss of information about the atmosphere in terms of the model parameters estimated in this work.

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